

AN-1990 Compensation for Current Mode Control SEPIC Converters

ABSTRACT

This application note discusses the use of SEPIC converters in various applications.

	Contents	
1	Introduction	2
2	Basic SEPIC Converters	2
3	Compensator Design	
4	Illustrative Example	4
5	Conclusion	(
6	Appendix A	7
7	Appendix B	8
	List of Figures	
1	A SEPIC Converter	2
2	A Compensator Implemented by a Transconductance Amplifier Circuit	;
3	Frequency Response of a Lag Compensator	
4	Frequency Response of the Un-Compensated System	!
5	Frequency Response of the Compensated System with 90° Phase Margin	(
	List of Tables	
1	Major Parameters of the Example SEPIC Converter	4
2	Parameters of the LM3478	!

All trademarks are the property of their respective owners.



Introduction www.ti.com

1 Introduction

SEPIC converters have a number of advantages. They allow an input voltage higher or lower than the output voltage. The input voltage and output voltage can be dc isolated by a capacitor. The use of a low side switch makes the switch driver easy to implement. Unlike buck-boost and Cuk converters, the output voltage of SEPIC converters is non-inverting. Hence, SEPIC converters are useful in many applications.

This application note presents the design of compensators for current mode control SEPIC converters. The LM3478 current mode controller will be used. Detailed procedures on designing a lag compensator will be presented in an illustrative example.

2 Basic SEPIC Converters

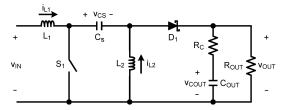


Figure 1. A SEPIC Converter

A SEPIC converter is shown in Figure 1. It consists of two inductors (L_1, L_2) and two capacitors (C_S, C_{OUT}) . Let v_{IN} and v_{OUT} be input and output voltages, v_{COUT} and v_{CS} be voltages across C_{OUT} and C_S , i_{L1} and i_{L2} be currents through L_1 and L_2 , and R_C be the equivalent series resistance (ESR) of C_{OUT} . Assume that the load is a resistor R_{OUT} , and that the switch S_1 and the diode D_1 are ideal.

In the continuous conduction mode (CCM), when S_1 is turned on, L_1 and L_2 are charged up by v_{IN} and v_{CS} respectively, while C_S and C_{OUT} are discharged by i_{L2} and the output current respectively. When S_1 is turned off, L_1 and L_2 are discharged, and C_S and C_{OUT} are charged up. The open loop small signal model of a SEPIC converter is

$$\widetilde{v}_{OUT} = \frac{N_d(s)\widetilde{d} + N_n(s)\widetilde{v}_{IN}}{\Delta(s)}, \tag{1}$$

where

 $N_d(s) = \overline{V}_{IN} R_{OUT} (1 + s R_C C_{OUT})$

$$\left[1 - \frac{s\bar{D}^2 L_1}{(1 - \bar{D})^2 R_{OUT}} + s^2 (L_1 + L_2) C_S - \frac{s^3 \bar{D} L_1 L_2 C_S}{(1 - \bar{D})^2 R_{OUT}}\right]$$
(2)

$$N_{n}(s) = \overline{D}(1-\overline{D})R_{OUT}(1+sR_{C}C_{OUT})\left(1+\frac{s^{2}L_{2}C_{S}}{\overline{D}}\right)$$
(3)

$$\Delta(s) = D_0 + D_1 s + D_2 s^2 + D_3 s^3 + D_4 s^4 \tag{4}$$

The coefficients of (4) will be listed in Appendix A. Also, $N_d(s)$ and $N_n(s)$ can be expanded to a polynomial as shown in Appendix A. The duty cycle d is the ratio between the on-time and the switching period T_{SW} of the switch S_1 . Its nominal value is

$$\overline{D} = \frac{\overline{V}_{OUT}}{\overline{V}_{IN} + \overline{V}_{OUT}}$$

Under current mode control, the current of S_1 , which is the sum of i_{L1} and i_{L2} , is fed to the controller during the on period in order to determine the on-time of S_1 . The small signal model of a current mode control SEPIC converter is

$$\widetilde{v}_{\text{OUT}} = \frac{N_{\text{cc}}(s)\widetilde{i}_{\text{C}} + N_{\text{cv}}(s)\widetilde{v}_{\text{IN}}}{D_{\text{cc}}(s)}$$
(5)

where i_C is the current control signal. It can be converted into a voltage control signal v_C by a resistor R_{SN} connecting between S_1 and the ground. Then the relationship between the output voltage and the voltage control signal can be formulated as follows:



www.ti.com Compensator Design

$$\widetilde{v}_{\text{OUT}} = \frac{N_{\text{cc}}(s)}{D_{\text{cc}}(s)R_{\text{SN}}} \widetilde{v}_{\text{C}}$$
(6)

where

$$D_{cc} = D_{c0} + D_{c1}S + D_{c2}S^{2} + D_{c3}S^{3} + D_{c4}S_{4} + D_{c5}S_{5} + D_{c6}S_{6},$$
(7)

$$N_{cc} = N_{c0} + N_{c1} s + N_{c2} s^2 + N_{c3} s^3 + N_{c4} s^4 + N_{c5} s^5 + N_{c6} s^6,$$
(8)

The coefficients of (7) and (8) will be shown in Appendix B.

3 Compensator Design

A compensator can be implemented by a transconductance amplifier, with an open loop gain of g_m and an output impedance of R_0 , connecting to a resistor R_{C1} and a capacitor C_{C1} in series to the ground, as shown in Figure 2. Let the negative input of the amplifier is connected to a reference voltage V_{REF} , and the positive input is connected to the output voltage v_{OUT} through a resistor divider network implemented by R_{E1} and R_{E2} , the transfer function relating v_C and v_{OUT} is

$$v_{C} = \left(\frac{R_{F2}}{R_{F1} + R_{F2}} v_{OUT} - V_{REF}\right) g_{m} \left[R_{0} / \left(R_{C1} + \frac{1}{C_{C1}}\right)\right]$$
(9)

By adding small signal perturbations, the AC equation can be obtained as follows:

$$\widetilde{v}_{C} = \frac{R_{F2}}{R_{F1} + R_{F2}} g_{m} R_{0} \frac{1 + s R_{C1} C_{C1}}{1 + s (R_{C1} + R_{0}) C_{C1}} \widetilde{v}_{OUT}.$$
(10)

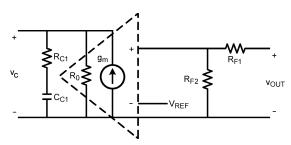


Figure 2. A Compensator Implemented by a Transconductance Amplifier Circuit

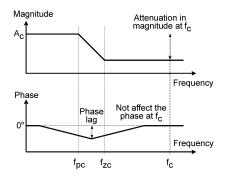


Figure 3. Frequency Response of a Lag Compensator

It can be shown from (10) that the compensator consists of a dc gain of A_C , and a pole and a zero located at frequencies f_{PC} and f_{ZC} . The three parameters can be formulated as

$$A_{C} = \frac{R_{F2}}{R_{F1} + R_{F2}} g_{m} R_{0},$$

$$f_{PC} = \frac{1}{2\pi (R_{C1} + R_{0}) C_{C1}},$$

$$f_{ZC} = \frac{1}{2\pi R_{C1} C_{C1}},$$
(11)



Illustrative Example www.ti.com

Since f_{PC} is always lower than f_{ZC} , (10) is a lag compensator. If R_{C1} is zero, (10) becomes a compensator with a dominant pole. The frequency response of the lag compensator is shown in Figure 3, the lag compensator provides an attenuation in magnitude at the high frequency. The degree of attenuation is determined by the distance between f_{PC} and f_{ZC} . It is because the magnitude is decreased at a slope of 20dB/decade between f_{PC} and f_{ZC} . The lag compensator also provides a phase lag. However, f_{PC} and f_{ZC} can be placed at a low frequency (much lower than the frequency of interest, e.g. the cross over frequency f_{C}) such that the lag compensator nearly does not affect the phase at the high frequency.

The aim of designing a lag compensator is to provide a desired phase margin for the compensated system. Starting from a bode plot of an un-compensated system, and a requirement of phase margin of Φ_m , a new f_C can be selected at the frequency corresponding to 180° - Φ_m of the un-compensated system. Then the magnitude of the un-compensated system at f_C can be found. The magnitude at f_C can be attenuated to 0dB by the lag compensator through proper design of f_{PC} and f_{ZC} . As a result, the compensated system will have a phase margin of Φ_m , and the cross over frequency will be f_C .

4 Illustrative Example

The design of a current mode control SEPIC converter with a nominal input voltage of 5V, an output voltage of 5V, and an output current of 0.5A will be shown. It is suitable for applications requiring a 5V output from four batteries, which can be 4.8V to 6V depending on whether 1.2V or 1.5V batteries are used. In this case, the input voltage may be higher or lower than the output voltage, and a SEPIC converter is a proper choice.

The major components of the SEPIC are listed in Table 1. A current mode controller LM3478 will be used. The parameters of the LM3478, which can be derived from the data sheet, are also listed in Table 2.

Other parameters of (6) are calculated below.

From (5),

 $\overline{D} = 0.5$

Also,

$$T_2 = \frac{T_{SW}}{2} = \frac{1}{2f_{SW}} \tag{12}$$

$$T_2 = 1.25 \,\mu s$$
 (13)

The parameter m_C is determined by an internal compensation ramp V_{SL} and an external compensation ramp determined by an internal current of 40 μ A passing through an external resistor R_{SL} . It can be calculated by the following equation:

$$m_{C} = (V_{SL} + 40 \ \mu A \ x \ R_{SL}) f_{SW} / R_{SN} = 3440000 A s^{-1}$$
 (14)

$$T_{M} = \frac{T_{SW}}{2} \left(2m_{C} + \frac{\overline{V}_{IN}}{L_{1}} + \frac{\overline{V}_{IN}}{L_{2}} \right)$$
= 8.979A (15)

Table 1. Major Parameters of the Example SEPIC Converter

Parameter	Value
∇_{IN}	5V
$ abla_{OUT}$	5V
R _{out}	10Ω
L ₁	33 µH
L ₂	33 µH
Cs	1 μF
C _{OUT}	100 μF
R _{COUT}	0.05Ω
f _{SW}	400 kHz
R _{SN}	0.02Ω
R _{SL}	2 kΩ



www.ti.com Illustrative Example

Table 2. Parameters of the LM34

Parameter	Value
V_{REF}	1.26V
g _m	800 μΩ ⁻¹
R_0	$A_V/g_m = 38/800 \ \mu\Omega^{-1} = 47.5 \ k\Omega$
V _{SL}	92 mV

Hence, all parameters for calculating the small signal model of (6) are obtained. A bode plot of (6) with the above parameters is shown in Figure 4.

Since \overline{V}_{OUT} and V_{REF} are 5V and 1.26V respectively, we can design that

$$R_{\rm F1} = 29.7 \text{ k}\Omega$$
 (16)

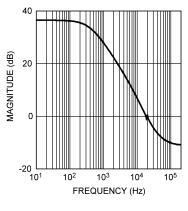
$$R_{F2} = 10 \text{ k}\Omega \tag{17}$$

From (10),

$$A_{C} = \frac{R_{F2}}{R_{F1} + R_{F2}} g_{m} R_{0}$$

$$= 9.57$$

$$= 19.62 dB$$
(18)



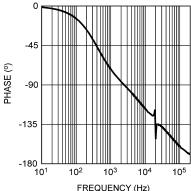


Figure 4. Frequency Response of the Un-Compensated System

In this example, a phase margin of 90° is desired. From Figure 4, the corresponding frequency (the frequency at which the phase is 180° - 90° = 90°) is 2.1 kHz (which will also be f_C of the compensated system), and the magnitude of the un-compensated system at 2.1 kHz is 21dB. This implies that the attenuation provided by the lag compensator is 21dB + A_C = 40.62 dB. Consequently, the distance between f_{PC} and f_{ZC} should be 2.031 decade (since the magnitude is 20dB/decade in between f_{PC} and f_{ZC}). To avoid affecting the phase at f_C , f_{ZC} is designed to be one decade before f_C , i.e. 210 Hz. Then f_{PC} should be 1.95 Hz. Hence,

$$1/R_{c1}C_{c1} = 2\pi \times 210 \text{ Hz}$$
 (19)



Conclusion www.ti.com

$$\frac{1}{(R_{C1} + R_0)C_{C1}} = 2\pi \times 1.95 \text{ Hz}$$

$$R_0C_{C1} = \frac{1}{2\pi \times 1.95 \text{ Hz}} - R_1C_{C1}$$

$$C_{C1} = 1.7 \mu\text{F}$$

$$R_{C1} = 445\Omega$$
(20)

Finally, select R_{C1} = 442 Ω and C_{C1} = 2.2 µF. The frequency response of the compensated system is shown in Figure 5. It can be found that the 0dB point is at around 2.5 kHz, and the phase margin is around 90°.

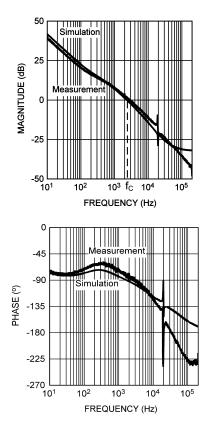


Figure 5. Frequency Response of the Compensated System with 90° Phase Margin

5 Conclusion

This application note details the design of a lag compensator for current mode control SEPIC converters operating in the continuous conduction mode. Based on the open loop bode plot, a lag compensator with 90° phase margin has been designed as an illustrative example. The design of compensator depends on a number of practical concerns including the requirement of transient response, robustness, and the effect of noise. Application engineers are suggested to design properly based on practical situations.



Appendix A www.ti.com

Appendix A 6

The coefficients of (4) are listed as follows.

$$\Delta(s) = D_0 + D_1 s + D_2 s^2 + D_3 s^3 + D_4 s^4, \tag{21}$$

$$D_0 = R_{OUT}(1-\overline{D})^2,$$

$$D_1 = L_M + (1-\overline{D})^2 R_C R_{OUT} C_{OUT},$$

$$D_2 = L_M(R_C + R_{OUT})C_{OUT} + (1 - \overline{D})^2(L_1 + L_2)R_{OUT}C_S$$

$$D_3 = L_1L_2C_S + (1 - \overline{D})^2(L_1 + L_2)R_CR_{OUT}C_SC_{OUT}$$

$$D_4 = L_1L_2(R_C + R_{OUT})C_SC_{OUT},$$

$$L_{M} = \overline{D}^{2}L_{1} + (1 - \overline{D})^{2}L_{2}. \tag{22}$$

From (2), N_d(s) can be expended as follows.

$$N_d(s) = N_0 + N_1 s + N_2 s^2 + N_3 s^3 + N_4 s^4,$$
(23)

$$N_0 = \overline{V}_{IN}R_{OUT}$$

$$N_1 = \overline{V}_{IN} R_C R_{OUT} C_{OUT} - \frac{\overline{D}^2}{\left(1 - \overline{D}\right)^2} \, \overline{V}_{IN} L_1 \,, \label{eq:N1}$$

$$N_2 = \overline{V}_{IN}(L_1 + L_2) R_{OUT} C_S - \frac{\overline{D}^2}{\left(1 - \overline{D}\right)^2} \, \overline{V}_{IN} L_1 R_C C_{OUT} \, ,$$

$$N_3 = \overline{V}_{\text{IN}}(L_1 + L_2) R_C R_{\text{OUT}} C_S C_{\text{OUT}} - \frac{\overline{D}}{\left(1 - \overline{D}\right)^2} \, \overline{V}_{\text{IN}} L_1 L_2 C_S \,, \label{eq:N3}$$

$$N_4 = -\frac{\overline{D}}{(1 - \overline{D})^2} \overline{V}_{IN} L_1 L_2 R_C C_S C_{OUT}.$$

From (3), $N_{n}(s)$ can be expended as follows.

$$N_n(s) = N_{n0} + N_{n1}s + N_{n2}s^2 + N_{n3}s^3$$

$$N_{n0} = \overline{D}(1 - \overline{D})R_{OUT}$$

$$N_{n1} = \overline{D}(1 - \overline{D})R_CR_{OUT}C_{OUT}$$

$$N_{n2} = (1 - \overline{D})L_2R_{OUT}C_S$$

$$N_{n3} = (1 - \overline{D})L_2R_CR_{OUT}C_SC_{OUT}.$$
 (25)

(24)



Appendix B www.ti.com

7 Appendix B

The coefficients of (7) are listed as follows.

$$\begin{split} D_{CC} &= D_{coS} S + D_{c_1} S + D_{c_2} S^2 + D_{c_3} S^3 + D_{c_4} S^4 + D_{c_5} S^5 + D_{c_5} S^6, \\ D_{c_0} &= C_{d_0} D_1 + C_{d_1} D_0 - C_{v_0} N_1 - C_{v_1} N_0, \\ D_{c_1} &= C_{d_0} D_2 + C_{d_1} D_1 + C_{d_2} D_0 - C_{v_0} N_2 - C_{v_1} N_1 - C_{v_2} N_0 \\ D_{c_2} &= C_{d_0} D_3 + C_{d_1} D_2 + C_{d_2} D_1 - C_{d_3} D_1 - C_{v_0} N_3 - C_{v_1} N_2 - C_{v_2} N_1 \\ D_{c_3} &= C_{d_0} D_4 + C_{d_1} D_3 + C_{d_2} D_2 - C_{d_3} D_1 - C_{v_0} N_4 - C_{v_1} N_3 - C_{v_2} N_2 \\ D_{c_4} &= C_{d_1} D_4 + C_{d_2} D_3 + C_{d_3} D_2 - C_{v_1} N_4 - C_{v_2} N_3 \\ D_{c_5} &= C_{d_2} D_4 + C_{d_3} D_3 - C_{v_2} N_4 \\ D_{c_6} &= C_{d_3} D_4 \\ C_{d_0} &= \frac{\overline{V}_{NL} L_1 L_2}{(1 - \overline{D})}, \\ C_{d_1} &= L_1 L_2 L_M T_M + \frac{\overline{D}}{(1 - \overline{D})} [(1 - D) L_2 - \overline{D} L_1 | \overline{V}_N L_1 \Big[T_2 + \frac{L_2}{R_{OUT}(1 - \overline{D})} \Big], \\ C_{d_2} &= \frac{\overline{V}_{NL} L_1 L_2}{(1 - \overline{D})} \Big[(L_1 + L_2) C_S - L_1 T_2 \frac{\overline{D}^2}{R_{OUT}(1 - \overline{D})} \Big], \\ C_{d_3} &= L_1^2 L_2^2 C_S T_M \\ C_{v_0} &= (1 - \overline{D}) L_1 L_2, \\ C_{v_1} &= \overline{D} L_1 (L_M - \overline{D} L_1) T_2, \\ C_{v_2} &= (1 - \overline{D}) L_1 L_2 (L_1 + L_2) C_S, \\ T_2 &= \frac{T_{SW}}{2}, T_{SW} \text{ is the switching period,} \\ T_M &= \frac{T_{SW}}{2} \Big(2 m_C + \frac{\overline{V}_N}{L_1} + \frac{\overline{V}_N}{L_2} \Big), \text{ m_c is the slope of a compensation ramp.} \end{aligned}$$

The coefficients of (8) are listed as follows.

coefficients of (6) are listed as follows.
$$N_{cc} = N_{c0} + N_{c1}s + N_{c2}s^2 + N_{c3}s^3 + N_{c4}s^4 + N_{c5}s^5 + N_{c6}s^6,$$

$$N_{c0} = C_{c0}N_0,$$

$$N_{c1} = C_{c0}N_1,$$

$$N_{c2} = C_{c0}N_2 + C_{c2}N_0,$$

$$N_{c3} = C_{c0}N_3 + C_{c2}N_1,$$

$$N_{c4} = C_{c0}N_4 + C_{c2}N_2,$$

$$N_{c5} = C_{c2}N_3,$$

$$N_{c6} = C_{c2}N_4,$$
 e

where

$$C_{c0} = L_1 L_2 L_M,$$

 $C_{c2} = L_1^2 L_2^2 C_S.$

8

IMPORTANT NOTICE

Texas Instruments Incorporated and its subsidiaries (TI) reserve the right to make corrections, enhancements, improvements and other changes to its semiconductor products and services per JESD46, latest issue, and to discontinue any product or service per JESD48, latest issue. Buyers should obtain the latest relevant information before placing orders and should verify that such information is current and complete. All semiconductor products (also referred to herein as "components") are sold subject to TI's terms and conditions of sale supplied at the time of order acknowledgment.

TI warrants performance of its components to the specifications applicable at the time of sale, in accordance with the warranty in TI's terms and conditions of sale of semiconductor products. Testing and other quality control techniques are used to the extent TI deems necessary to support this warranty. Except where mandated by applicable law, testing of all parameters of each component is not necessarily performed.

TI assumes no liability for applications assistance or the design of Buyers' products. Buyers are responsible for their products and applications using TI components. To minimize the risks associated with Buyers' products and applications, Buyers should provide adequate design and operating safeguards.

TI does not warrant or represent that any license, either express or implied, is granted under any patent right, copyright, mask work right, or other intellectual property right relating to any combination, machine, or process in which TI components or services are used. Information published by TI regarding third-party products or services does not constitute a license to use such products or services or a warranty or endorsement thereof. Use of such information may require a license from a third party under the patents or other intellectual property of the third party, or a license from TI under the patents or other intellectual property of TI.

Reproduction of significant portions of TI information in TI data books or data sheets is permissible only if reproduction is without alteration and is accompanied by all associated warranties, conditions, limitations, and notices. TI is not responsible or liable for such altered documentation. Information of third parties may be subject to additional restrictions.

Resale of TI components or services with statements different from or beyond the parameters stated by TI for that component or service voids all express and any implied warranties for the associated TI component or service and is an unfair and deceptive business practice. TI is not responsible or liable for any such statements.

Buyer acknowledges and agrees that it is solely responsible for compliance with all legal, regulatory and safety-related requirements concerning its products, and any use of TI components in its applications, notwithstanding any applications-related information or support that may be provided by TI. Buyer represents and agrees that it has all the necessary expertise to create and implement safeguards which anticipate dangerous consequences of failures, monitor failures and their consequences, lessen the likelihood of failures that might cause harm and take appropriate remedial actions. Buyer will fully indemnify TI and its representatives against any damages arising out of the use of any TI components in safety-critical applications.

In some cases, TI components may be promoted specifically to facilitate safety-related applications. With such components, TI's goal is to help enable customers to design and create their own end-product solutions that meet applicable functional safety standards and requirements. Nonetheless, such components are subject to these terms.

No TI components are authorized for use in FDA Class III (or similar life-critical medical equipment) unless authorized officers of the parties have executed a special agreement specifically governing such use.

Only those TI components which TI has specifically designated as military grade or "enhanced plastic" are designed and intended for use in military/aerospace applications or environments. Buyer acknowledges and agrees that any military or aerospace use of TI components which have *not* been so designated is solely at the Buyer's risk, and that Buyer is solely responsible for compliance with all legal and regulatory requirements in connection with such use.

TI has specifically designated certain components as meeting ISO/TS16949 requirements, mainly for automotive use. In any case of use of non-designated products, TI will not be responsible for any failure to meet ISO/TS16949.

Products Applications

Audio www.ti.com/audio Automotive and Transportation www.ti.com/automotive Communications and Telecom **Amplifiers** amplifier.ti.com www.ti.com/communications **Data Converters** dataconverter.ti.com Computers and Peripherals www.ti.com/computers **DLP® Products** www.dlp.com Consumer Electronics www.ti.com/consumer-apps

DSP **Energy and Lighting** dsp.ti.com www.ti.com/energy Clocks and Timers www.ti.com/clocks Industrial www.ti.com/industrial Interface interface.ti.com Medical www.ti.com/medical logic.ti.com Logic Security www.ti.com/security

Power Mgmt power.ti.com Space, Avionics and Defense www.ti.com/space-avionics-defense

Microcontrollers <u>microcontroller.ti.com</u> Video and Imaging <u>www.ti.com/video</u>

RFID www.ti-rfid.com

OMAP Applications Processors www.ti.com/omap TI E2E Community e2e.ti.com

Wireless Connectivity <u>www.ti.com/wirelessconnectivity</u>